Report of
Advanced Functional Analysis and its Applications

Conducted by
Dept. of Mathematics
IIT Hyderabad during Nov 25-29, 2019

Funded by MHRD scheme on
TEQIP (Phase III)

Coordinated by
Dr. Tanmoy Paul
Department of Mathematics
Indian Institute of Technology Hyderabad
http://math.iith.ac.in/Tanmoy.html

1. Overview

Functional Analysis is one of the core areas in Analysis. Several Universities and Institutes offer this subject in their Master's level as well as in their research level. This subject can be taught from many angles depending on the requirement of this subject. Functional Analysis, in its wide sense, includes the study of various aspects of topologies on Vector spaces, stochastic theory, non-commutative harmonic analysis, but its core is the study of normed spaces, together with the study of Function spaces over the various domain, behavior of the operators on normed spaces both from the linear and nonlinear point of view. Even people who are studying Mathematical Economics, Financial Mathematics, Actuarial Science, Electrical, Mechanical Engineering are also using this subject and its various tools in their respective fields.

2. Objectives

Our prime objective is to offer some advanced topics in this field which would be helpful for a Lecturer and also a research scholar. It is always important and helpful for a Lecturer having a wide range of knowledge in the respective subject and its allied fields. We believe people will get a deeper view and exposure to the subject on attending this workshop.

3. Speakers of the workshop and the TOTal duration of the lectures delivered

(1) *Dr. Anil Karn* (AK)(4.5 Hours): is an Associate professor from NISER Bhuboneswar. He joined NISER on August, 2011 as a Reader-F.

(2) *Dr. Gadadhar Misra* (GM)(4.5 Hours): completed his PhD from the State University of New York at Stony Brook. He was a Visiting Assistant Professor at University of Georgia and the University of California. Started his career in Indian Statistical Institute and moved to the IISc on 2008. He was selected for the J C Bose Fellowship in the year of 2008, awarded the presgious S. S. Bhatnagar Prize (CSIR) in 2001 and Biju Patnaik Award for his Scientific Excellence from the Odisha Bigyan Academy in 2013. His research is mostly related to operator theory which involves finding curvature inequalities for operators in the Cowen-Douglas class, describing homogeneous vector bundles and study of Hilbert modules.

(3) *Dr. M. N. Namboodiri* (MNN)(4.5 Hours): was a professor of CUSAT, retired on 2014. He taught at IIT Palkad for a semester and joined CUSAT as an Emeritus Professor. Currently he is an Emeritus Scientist at CUSAT. His area of specialization is Functional Analysis and work on problems related to spectral analysis of operators, Chebyshev subspaces of operator algebras, Korovkin and Fellertype limit theorems in operator algebras.

(4) *Dr. Amin Sofi* (AS)(4.5 Hours): is an Adjunct Faculty at the Central University of Kashmir, Srinagar. Earlier he completed his term as Emeritus Professor at Kashmir University, Srinagar. He was awarded Ramanujan Prize in 2009. His research interests is Functional Analysis, in particular, Finite dimensionality phenomena, Embeddings theorems, Extendability of maps in Banach spaces.

(5) *Dr. T. S. S. R. K. Rao* (TSS)(4.5 Hours): is a Visiting Professor from Ashoka University. He is a former Head of ISI Bangalore and Professor in Charge of ISI. Prof. Rao is one of the eminent Functional Analyst in India. He worked in various domain in it like Approximation Theory, Analysis on Banach spaces and their geometry, Choquet Theory, Optimization techniques etc.

(6) *Dr. Tirthankar Bhattacharyya* (TB)(4.5 Hours): is a professor in the Dept. of Mathematics of IISc. His area of research includes Hilbert space and Operator theory. His research has been widely appreciated all over the world and he has many collaborators in India and abroad. He is passionate in his teaching and has consistently been rated as an excellent teacher by the students of IISc as well as students in many summer courses and short term courses which he has participated in elsewhere.

(7) *Dr. Tanmoy Paul* (TP)(4.5 Hours): is an Assistant Professor from IIT Hyderabad. He completed his PhD from ISI Kolkata, his PhD work is related to Functional Analysis, Analysis on Banach spaces. His current area of interest is Functional Analysis, Banach space theory, Convex analysis and Geometric measure theory.

4. Coordinator of the event

Dr. Tanmoy Paul: Assistant Professor, Dept. of Mathematics, IIT Hyderabad.

5. COURSE DETAILS

- Duration of the event: November 25th to 29th.
- Lecture Schedule:
  (1) **Day 1:**

  Lecture 1(TSS) : An important aspect of operator theory is the study of representing of operators in terms of measures. Starting with the classical Riesz representation theorem on the space of continuous functions, in these lectures we gave an exposition of representation of operators.

  Lecture 2(TB) : Discussed the basics of dilation theory, positive definite kernels and von Neumann inequality. Made the basis for advanced topics.

  Lecture 3(TSS) : This approach allows us to understand operator theoretic properties better. Weakly compact operators on space of continuous functions Have a representation theorem due to Battle-Dunford and Schwartz. In the first lecture we recalled this set up and exhibited the duality of operators and Measures.

  Lecture 4(TP) : In Lecture 1, some basic properties of convex functions are discussed. The notion of subdifferential of a convex function is introduced. All the notions are extended to infinite dimensional Banach spaces.

  Lecture 5(TP) : In Lecture 2, smoothness and strict convexity is introduced. Some basic renorming are discussed. It is observed that any separable Banach space always admit an equivalent strictly convex and Locally Uniformly Convex renorming.

  (2) **Day 2:**

  Lecture 1(TSS) : A well known class of Banach spaces initiated by Lindenstrauss is those spaces for which the bridal is a space of continuous functions. Since for a weakly compact operator the bi-adjoint is also weakly compact, these classes are ideally suited for extending the classical set up.

  Rest of the lectures were devoted to the Rao-Roy extension theorem for separable Lindenstrauss spaces. Several applications of this were given.

  Lecture 2(TB) : De Branges-Rovnyak construction of dilation for a pure contraction. Introduction to multivariable dilation.

  Lecture 3(AK) : Introduced the notion and properties of vector lattices. I also discussed the dual of a vector lattice.

  Lecture 4(MNN) Classical Choquet Theory is discussed. Saskin's result related to Choquet boundary is discussed with its proof. Korovkin's Theorem on the space $C[0, 1]$ is proved. As a motivation of this result *Korovkin set* is defined.

  (3) **Day 3:**

  Lecture 1(TB) : Discussed in detail some parts of Das, B. Krishna; Sarkar, Jaydeb *Ando dilations, von Neumann inequality, and distinguished varieties*. J. Funct. Anal. 272 (2017), **no. 5**, 2114−2131.

  Lecture 2(GM) : The polynomial calculus for a commuting tuple was described. The Von-Neumann inequality and the Sz.-Nagy–Foias unitary power dilation were discussed. The relationship of these notions with complete contractivity was discussed.

  Lecture 3(AS) : Some basic results are discussed: Goldstein Theorem, Banach-Alaoglu Theorem, Krein-Milman Theorem are discussed.

  Lecture 4(MNN) : Aiming for a non commutative analogue of Saskin's result and Korovkin's Theorem the Gelfand-Norman-Siegel Theorem is proved. Choquet boundary is introduced in its non commutative analogue.

  (4) **Day 4:**

  Lecture 1(GM) : It was pointed out that (a) for commuting a pair of contractions, the von Neumann inequality is valid via the Ando dilation theorem, (b) while for a commuting tuple of contractions, these two notions are distinct, (c) in general, there is neither any von Neumann inequality (Varopolous example) nor a dilation theorem (Parrott's example) commuting tuple of contractions.

  Lecture 2(AK) : Discussed normed vector lattices and Banach lattices and their duals. I also introduced the notion of unital AM-spaces.

  Lecture 3(AS) : Notions related to the Lipschitz functions are discussed. Properties of Lipschitz functions defined over a Banach space are discussed.

Lecture 4(MNN) : Possible generalizations of Choquet's results in their non commutative analogue are discussed. A few recent developments are discussed.

(5) Day 5:

Lecture 1:(GM) : The relationship between some of these notions with Grothendieck inequality was discussed. A proof of the Grothendieck inequality was indicated and applications were given.

Lecture 2:(AK) : discussed the geometric properties of the pure state space of a unital AM-space. Finally, I completed the proof of Kakutani Theorem.

Lecture 3(AS) : The notion of Lipschitz free Banach space is introduced. Free space of a metric space is introduced. Unsolved problems related to Lipschitz functions are discussed.

Lecture 4(TP) : It is observed that a Banach space whose dual is separable has property that any continuous convex function is Frechet differentiable in a dense G_δ set. The notion of *Asplund* space is introduced and its various characterizations are discussed.

6. ATTENDED PARTICIPANTS

(1) Ph.D. students working in the area of theoretical and applied mathematics whose works are related with the structures of Analytic functions defined on both finite and infinite dimensional spaces, Operator Theory, Operator spaces.

(2) Post-doctoral fellows in Pure and Applied Mathematics.

(3) Both from TEQIP and non TEQIP funded College/ University lecturers who are interested to pursue their research career in Analysis.

7. EVENT SCHEDULE

Day	Teaching slot 9.30–11		Teaching slot 11.15–12.45		Teaching slot 14–15.30		Teaching slot 15.45–17.15	Teaching slot 17.15–18.45
Day 1	T.S.S.R.K Rao	Tea break	Tirthankar Bhat-tacharyya	Lunch Break	T.S.S.R.K Rao	Tea Break	Tanmoy Paul	Tanmoy Paul
Day 2	TSSRK Rao	Tea break	Tirthankar Bhat-tacharyya	Lunch Break	Anil Karn	Tea break	Namboodi	iNo Teaching slot
Day 3	Tirthankar Bhat-tacharyya	Tea break	Gadadhar Misra	Lunch break	Amin Sofi	Tea break	Namboodi	iNo Teaching slot
Day 4	Gadadhar Misra	Tea break	Anil Karn	Lunch break	Amin Sofi	Tea break	Namboodi	iNo Teaching slot
Day 5	Gadadhar Misra	Tea break	Anil Karn	Lunch break	Namboodi	iTea break	Tanmoy Paul	No Teaching slot