Structural dynamic simulations using Newmark-beta integrator

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1. Introduction

Structural engineers are familiar with the analysis of structures for static loads in which a load is applied to the structure and a single solution is obtained for the resulting displacements and member forces. When considering the analysis of structures for dynamic loads, the term dynamic simply means time-varying. Hence, the loading and all aspects of the response vary with time. This results in possible solutions at each instant during the time interval under consideration. From an engineering standpoint, the maximum values of the structural response are usually the ones of particular interest, especially when considering the case of structural design.

Most structural systems will be subjected to some form of dynamic loading during their lifetime. The sources of these loads are many and varied. The ones that have the most effect on structures can be classified as environmental loads that arise from winds, waves, and earthquakes. A second group of dynamic loads occurs as a result of equipment motions that arise in reciprocating and rotating machines, turbines, and conveyor systems. A third group is caused by the passage of vehicles and trucks over a bridge. Blast-induced loads can arise as the result of chemical explosions or breaks in pressure vessels or pressurized transmission lines. Most of the dynamic models arising in practice cannot be completely solved by analytic techniques; thus, numerical simulations are of fundamental importance in gaining an understanding of dynamical systems. It is therefore crucial to understand the behavior of numerical simulations of dynamical systems in order to interpret the data obtained from such simulations and to facilitate the design of algorithms which provide correct qualitative information without being unduly expensive. These two concerns lead to the study of the convergence and stability properties of numerical methods for dynamical systems.

There are various numerical tools available to carryout the dynamic analysis on structures. Among them Runge–Kutta method [1], the central difference method, finite difference method, Newmark-beta integrator[2] are a few of them[3]. In this work Newmark-beta integrator has been used to study the dynamics of 2 degree of freedom system.

1.1. Newmark-beta method

Newmark-beta integration scheme the displacements and velocities are approximated using the following assumptions:

$$\dot{\mathbf{U}}^{t+\Delta t} = \dot{\mathbf{U}}^{t} + \left[(1-\delta)\ddot{\mathbf{U}}^{t} + \delta\ddot{\mathbf{U}}^{t+\Delta t}\right]\Delta t \tag{1}$$

$$\mathbf{U}^{t+\Delta t} = \mathbf{U}^{t} + \dot{\mathbf{U}}^{t}\Delta t + \left[\left(\tfrac{1}{2}-\alpha\right)\ddot{\mathbf{U}}^{t} + \alpha\ddot{\mathbf{U}}^{t+\Delta t}\right]\Delta t^2 \tag{2}$$

Here, α and δ are the parameters that can be determined to obtain integration accuracy and stability. In this work $\delta = \frac{1}{2}$ and $\alpha = \frac{1}{4}$ are considered, which corresponds to constant-average-acceleration method that provides unconditionally stable solution. In addition to the above equations for solution of the displacements, velocities, and acceleration at time $t + \delta t$, the equilibrium equations at time $t + \delta t$ are also used as follows:

$$\mathbf{M}\ddot{\mathbf{U}}^{t+\Delta t} + \mathbf{C}\dot{\mathbf{U}}^{t+\Delta t} + \mathbf{K}\mathbf{U}^{t+\Delta t} = \mathbf{R}^{t+\Delta t} \tag{3}$$

Considering the trapezoidal rule, which is mostly used, solving for $\ddot{\mathbf{U}}^{t+\Delta t}$ and $\dot{\mathbf{U}}^{t+\Delta t}$ in terms of $\mathbf{U}^{t+\Delta t}$, we solve for each time step as follows:

$$\left(\frac{4}{\Delta t^2}\mathbf{M} + \frac{2}{\Delta t}\mathbf{C} + \mathbf{K}\right)\mathbf{U}^{t+\Delta t} = \mathbf{R}^{t+\Delta t} + \mathbf{M}\left(\frac{4}{\Delta t^2}\mathbf{U}^{t} + \frac{4}{\Delta t}\dot{\mathbf{U}}^{t} + \ddot{\mathbf{U}}^{t}\right) + \mathbf{C}\left(\frac{2}{\Delta t}\mathbf{U}^{t} + \dot{\mathbf{U}}^{t}\right) \tag{4}$$

2. Result

In-order to apply the explained Newmark-beta integration method to a dynamical system, a two degree of freedom system spring-mass system with the following governing equation is considered:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2 \end{Bmatrix} \tag{5}$$

Equation 5 is solved using the Newmark-beta integration method and the result is shown in Fig. 1. In-order to validate the result obtained the Eq. 5 has also been solved using Runge–Kutta 4th order technique and Matlab inbuilt solver ode45, and the corresponding results re also shown in Fig. 1. From the results it is observed that the results obtained from the Newmark-beta integration method matches very well with the RK4 method and Matlab ode45 solver.

3. Conclusion

The Newmark-beta Integrator used in structural dynamics is adapted in this work and is mentioned in this report for numerical solution of differential equations of systems with multiple degrees of freedom. Newmark-beta Integrator allows the direct solution of the second- order differential equation or a system of second-order differential equations without the need for the transformation to a pair of simultaneous first-order differential equations.

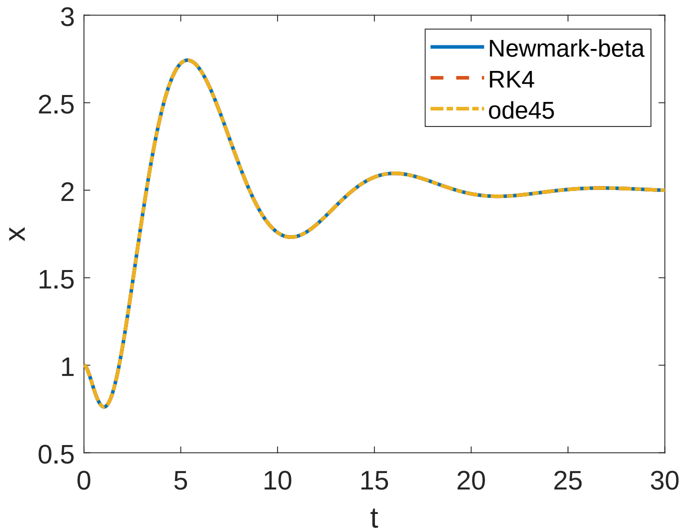

Figure 1: Solution for the spring-mass-damper system shown in Eq. 5 using Newmark-beta, RK4 method and ode45.

4. References

[1] E. Hairer, C. Lubich, M. Roche, The numerical solution of differential-algebraic systems by Runge-Kutta methods, Vol. 1409, Springer, 2006.

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[3] M. Dokainish, K. Subbaraj, A survey of direct time-integration methods in computational structural dynamicsi. explicit methods, Computers & Structures 32 (6) (1989) 1371–1386.