Stability Analysis of Turning

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1. Introduction

Chatter in machining processes like turning, milling and drilling is not induced by external periodic forces, but by instability in the cutting force [1, 2]. The disturbance caused by a wavy work surface on the tool motion influences the cutting tool dynamics and leads to tool vibration. This tool vibration generates new waviness on the work surface and leads to increased tool vibration with higher amplitudes. This regenerative vibration (positive feedback) leads to instability in the machining process and creates chatter. In 1907, Taylor [**?**] found that the steady-state cutting force depends on the chip width w, tool feed velocity v, and the chip thickness f. Taylor formulated a simple nonlinear model for cutting force F:

$$F = Kwf^n, \tag{1}$$

where K is the cutting coefficient in the feed direction and n is a constant that depends on the material [3].

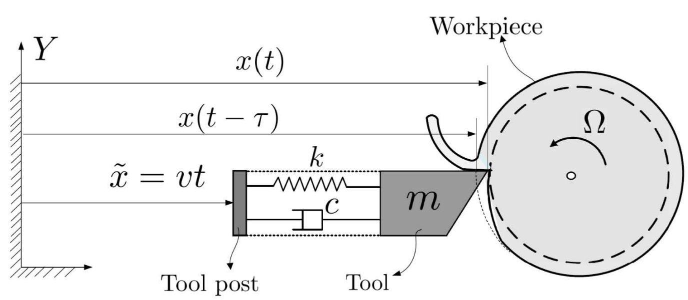

Figure 1: Schematic of single-degree-of-freedom orthogonal turning. The tool post moves with a constant velocity v and its position is represented by $\tilde{x}$. The tool tip position at current time is at $x(t)$ and its position before one revolution was at $x(t\tau)$.

Figure **??** shows a schematic of a single-degree-of-freedom (SDoF) model of orthogonal turning. It is assumed that the work-piece is revolving at a rate of against a flexible tool. The tool has a mass of m, damping coefficient c, stiffness k, and an un-sprung length of l. The tool is mounted on a tool post which moves at a feed velocity of v with respect to an inertial frame. The variable $x(t)$ is the position of the tool tip measured from the inertial frame at time t, and $x(t\tau)$ is the tool position at a previous time $(t\tau)$. Here, τ is the time for one revolution of the work-piece. We write the mathematical description of the SDoF turning model as follows:

$$m\ddot{x}(t) = -c(\dot{x}(t) - v) - k(x(t) - vt - l) - \Delta F, \tag{2}$$

where ΔF is the cutting force, which can be obtained from Eq. 1:

$$\Delta F = Kw(x(t) - x(t - \tau))^n. \tag{3}$$

Here, $(x(t) - x(t - \tau))$ is the dynamic chip thickness due to tool vibration. In this section, we are deriving a governing equation for SDoF orthogonal turning. We now introduce the following transformation:

$$x(t) - \xi(t) + vt + C. \tag{4}$$

Here, $\xi(t)$ represents a small perturbation around the static equilibrium, and C is a constant. Upon substituting Eq. 4 and Eq. 3 into Eq. 2 and rearranging, we obtain the following second-order DDE representing the dynamics of the perturbation ξ:

$$m\ddot{x}i + c\dot{\xi} + k\xi = -k(C - l) - Kw(\xi(t) - \xi(t - \tau + v\tau))^n. \tag{5}$$

Expanding $(\xi(t)\xi(t\tau) + v\tau)^n$ using a Taylor series, neglecting the higher-order terms, and selecting $C = (Kw(v\tau)^n + kl)/k$ in Eq. 5, we arrive at the following equation [4]:

$$m\ddot{\xi} + c\dot{\xi} + k\xi = -Kw(v\tau)^{n-1}(\xi(t) - \xi(t - \tau)). \tag{6}$$

Now, we substitute $c = 2m\omega_n\xi$ into Eq. 6, where $\omega_n = pk/m$ is the natural frequency of the cutting tool:

$$\ddot{\xi} + 2\zeta\omega_n\dot{\xi} + \omega_n^2\xi = -\tilde{\omega}(\xi(t) - \xi(t - \tau)). \tag{7}$$

Here, $\tilde{\omega} = Kw(v\tau)^{n-1}/(m\omega_n^2)$ is the normalized depth of the cut.

To improve the stability (chatter margin) of the turning process, Segalman and Butcher [5] proposed an impedance modulation approach. In this approach, the mechanical stiffness k of the tool is made time periodic by adding an oscillatory stiffness $km \cos(\gamma t)$. By defining $\alpha = km/k$ and $\ = \gamma/\gamma n$, the non-dimensional equation governing the dynamics of the tool can be written as:

$$\ddot{\xi} + 2\zeta\omega_n\dot{\xi} + \omega_n^2(1 + \alpha\cos(kt))\xi = -\tilde{\omega}(\xi(t) - \xi(t - \tau)). \tag{8}$$

Equation 8 is a DDE with a time-periodic coefficient. Another approach that is commonly employed to improve the stability of turning is to introduce a time-periodic delay and is achieved by spindle-speed modulation [**?**]. Here, a cosine-type spindle speed modulation is used, and is represented by $\Omega(t) = \Omega_0 + \Omega_1\cos(\omega_m t)$ where Ω_0 is the mean spindle speed. Ω_1 is the amplitude of the spindle speed, and ω_m is the angular modulation frequency. When $\Omega_1 < 0.2\Omega_0$, then the time delay [6] can be expressed as follows:

$$\tau(t) = \tau_0 - \tau_1\cos(\omega_m t). \tag{9}$$

The dynamical model of such a turning process with varying spindle speed is shown below [6]:

$$\ddot{\xi}(t) + 2\zeta\dot{\xi}(t) + \xi(t) = -\tilde{\omega}(\xi(t) - \xi(t - \tau(t))), \tag{10}$$

where $\tilde{\omega} = Kw/(m\omega_n^2)$ is the normalized depth of the cut. Equation10 is a DDE with time-periodic delay. By considering the distributed force between the tool and the workpiece, a more general model for turning was proposed by Khasawneh et al. [7]. The governing equation in this model takes the following form:

$$\ddot{\xi} + 2\zeta\dot{\xi}(t) + \omega_n^2\xi(t) = -\tilde{\omega}\int_0^{ts}(\xi(t-\hat{t}) - \xi(t-\tau(t)-\hat{t}))w(\hat{t})d\hat{t}, \tag{11}$$

where t_s is the chip contact time on the tool. In Eq. 11, $w(\hat{t})$ is the kernel function and contains the information of the force distribution on the tool. When $w(\hat{t}) = \delta(\hat{t})$ we recover Eq.7.

From this, we can see that depending on the physical process and modeling choice, the turning problem can be represented as a DDE with constant coefficients and delays (Eq.7), with time-periodic coefficients and constant delays (Eq.8), or with constant coefficients and time-periodic delays (Eq. 10), or with constant coefficients and distributed delays (Eq. 11).

2. Result

To study the stability, the following governing equation of motion for turning with 1 DoF is considered:

$$\ddot{x} + 2\xi\omega\dot{x} + \omega^2 x(t) = -\frac{\omega h(t)}{m}(x(t) - x(t-\tau)) \tag{12}$$

Upon solving the above equation using semi-desensitization method the following stability chart is obtained. In the Fig. 2 the black line shows the stability boundary below which the system will be stable and unstable

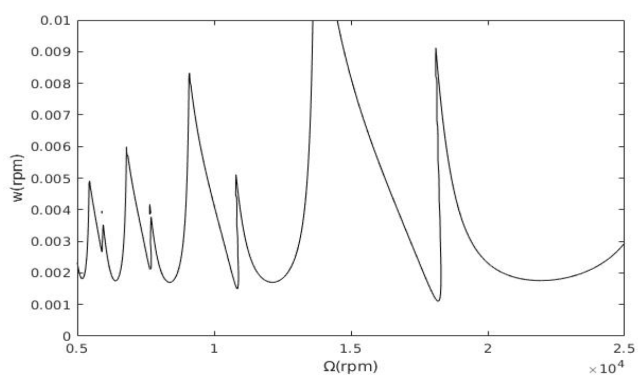

Figure 2: Schematic of single-degree-of-freedom orthogonal turning. The tool post moves with a constant velocity v and its position is represented by $\tilde{x}$. The tool tip position at current time is at $x(t)$ and its position before one revolution was at $x(t\tau)$.

in the above portion.

3. Conclusion

Turning is one of the machining process and disturbance caused by a wavy work surface on the tool motion influences the cutting tool dynamics and leads to tool vibration. In this work the governing equations of motion for turning are derived and represented in the form of a delay differential equations. Then the DDEs are solved using semi-desensitization technique and the corresponding stability chart is reported.

4. References

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